

MAKERERE UNIVERSITY

COLLEGE OF COMPUTING AND INFORMATION SCIENCES

SCHOOL OF COMPUTING & INFORMATICS TECHNOLOGY

END OF SEMESTER II EXAMINATION 2021/2022

PROGRAMME: BCSC (YR I) and BSSE (YR 2)

YEAR OF STUDY: I & II

COURSE NAME: CALCULUS I

COURSE CODE: MTH 1203

DATE: 02nd September 2022

TIME: 4:00-7:00PM

EXAMINATION INSTRUCTIONS

1. ATTEMPT ALL QUESTIONS IN SECTION A (40 MARKS)
2. ATTEMPT THREE (03) QUESTIONS IN SECTION B (60 MARKS)
3. DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO
4. ALL ROUGH WORK SHOULD BE IN YOUR ANSWER BOOKLET
5. SHOW ALL THE NECESSARY WORKINGS

SECTION A: (40 marks)

- a. Find $\lim_{x \rightarrow \infty} \frac{x^4 - 2x^2 + 3x - 4}{4x^4 - 4}$ (4 marks)
- b. A spring has a natural length of 18 inches and a force of 20 lbs. is required to stretch and hold the spring to a length of 24 inches. What is the work required to stretch the spring from a length of 21 inches to a length of 26 inches? (5 marks)
- c. Consider the function $h(x) = \frac{e^x + e^{-x}}{2}$ for $-\infty < x < \infty$. Find the interval(s) on which $h(x)$ is increasing. (5 marks)
- d. Find the equation of the tangent line to the curve of $x^2 + (y - 3)^3 = 9$ at $x = 1$. (5 marks)
- e. Evaluate the following Integrals.
- i. $\int_0^\pi (x + \sin(x)) dx$ (3 marks)
- ii. $\int_0^2 \sqrt{4 - x^2} dx$ (4 marks)
- f. State the three conditions that should be met for a function to be continuous. (3 marks)
- g. Determine if the following function is continuous or discontinuous at $x = -6$. (4 marks)
- $$g(x) = \begin{cases} 1 - 3x & x < -6 \\ 7 & x = -6 \\ x^3 & -6 < x < 1 \\ 1 & x = 1 \\ 2 - x & x > 1 \end{cases}$$
- h. Given $f(t) = t^2 + 3$, find
- i. $f^{-1}(t)$. (3 marks)
- ii. The domain and range of $f^{-1}(t)$. (4 marks)

SECTION B: (60 MARKS)

QUESTION 1:

- a. Suppose that $s(t) = 4t\sqrt{t} + \sqrt{t}$ is the position function of a particle, where s is in meters and t is in seconds. Find the particle's instantaneous velocity at time $t = 4$ s.
(6 marks)
- b. The position of an object at any time is given by $s(t) = 3t^4 - 40t^3 + 126t^2 - 9$. Determine all the points where the object is not moving.
(7 marks)
- c. Given the following functions, find $\frac{dy}{dx}$.
- i. $y = (x^2 - x)^{\cos 2x}$ (3 marks)
- ii. $\sin y + x^2y^3 - \cos x = 2y$ (4 marks)

QUESTION 2:

- a. Evaluate the following limits.
- i. $\lim_{x \rightarrow 5} \frac{(\sqrt{x^2 - 9}) - 4}{x^2 - 6x + 5}$ (5 marks)
- ii. $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos(2x)}$ (5 marks)
- b. Determine constants a, b, c, d so that the function below is continuous at $x = -1$ and differentiable at $x = 1$.
(10 marks)

$$g(x) = \begin{cases} \frac{x^2 + ax + 7}{x^2 + 4x + 3} & \text{if } x < -1 \\ b & \text{if } -1 \leq x \leq 1 \\ e^{c(x-1)} + dx & \text{if } 1 < x \end{cases}$$

QUESTION 3:

- a. Find the absolute maximum and the absolute minimum values of $f(x) = (x^2 - 3)e^x$ on the interval $[-2, 2]$.
(6 marks)
- b. Determine $\frac{dy}{dx}$ in terms of x and y for the equation $x + xy^2 - y = 3$.
(5 marks)
- c. Identify all the intervals where the function $f(x) = x^3 - 6x^2 + 5x + 1$ is concave down and find the inflection point(s), if any.
(5 marks)
- d. Differentiate the function $y = 6 + 4\sqrt{x} \operatorname{cosec}(x)$ with respect to x .
(4 marks)

QUESTION 4:

- a. By sketching the curves, determine the area of the region bounded by the equation $y = x^2 - 4x$ and $y = 6 - 3x$. (7 marks)
- b. Determine the value of $\int_2^9 f(x)dx$ given that $\int_5^2 f(x)dx = 3$ and $\int_5^9 f(x)dx = 8$. (3 marks)
- c. Find the average value of $f(x) = 4x^2 - x + 5$ on $[-2, 3]$ and determine the value(s) of c that satisfies the Mean Value Theorem for Integrals of $f(x)$ in the interval $[-2, 3]$. (7 marks)
- d. Find the following integral. (3 marks)

$$\int x \sin(x^2) \cos(x^2) dx$$

QUESTION 5:

- a. Solve for $2\cos^2 x - \sqrt{3} \cos x = 0$ on $0 \leq x \leq 2\pi$ (6 marks)
- b. Find all the solutions to the following equations. If there are no solutions, clearly explain why.
- i. $\log w + \log(w - 21) = 2$ (3 marks)
- ii. $4x + 1 = (12x + 3)e^{x^2 - 2}$ (4 marks)
- c. Determine $h(t)$ given that $h''(t) = 24t^2 - 48t + 2$, $h(1) = -9$ and $h(-2) = -4$. (7 marks)

"END"